 

# Instituting seminal teaching-datums: examples from plate and shell theory

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ABSTRACT: As engineering courses are applications-based for the most part, it is not surprising that many educators delve directly into applying the subject matter from the very first day of instruction without first imparting to students the essential reference points required to build a more seamless understanding of the new topic. In this article, the authors explore an engineering class in the area of mechanics – namely, plates and shells – and illustrate a subset of the pertinent background materials – three, in particular, herein referred to as *seminal teaching-datums* – that are argued to provide vital contributions to the overall experience and efficacy of learning comprehension.

## INTRODUCTION

As opposed to initiating, a swim, wherein the experienced athlete has learned that the best way to get acclimated to the water is by simply diving right in, teaching – if it is to be especially meaningful and effective – often requires the instructor to proceed gradually. This is particularly true in engineering courses that, by their very nature, involve the application of physics and mathematics towards the solution of a wide variety of problems. As such, they have the tendency of diverting the instructor away from placing adequate emphasis on important background details when introducing a new subject. Therefore, it is not uncommon for engineering students to perform mathematical operations by rote without comprehending the practical rationale that was responsible for their derivations in the first place. A good example of this is observed among engineering students who routinely use commercially-available finite element software programs to solve problems and fail to grasp the practical rationale behind the analyses at the expense of having to perform any mathematical operations on their own.

An effective countermeasure for the teacher in attempting a solution of such a predicament is to use what will herein be referred to as *teaching-datums*. To introduce this concept by analogy, consider the simple task of determining the centroid of a geometric area (ie the point at which the area can be concentrated to render an invariant first moment about any axis [1]), as required in the sophomore-level engineering mechanics of materials class.

In a conventional *xy*-Cartesian coordinate system, the vertical distance of the centroid is typically denoted as $\bar{y}$ and is equal to the ratio of the first moment-of-area about the *x* axis to the total area of the region in question; similarly, the horizontal distance is denoted as $\bar{x}$. The key point to bear in mind about $\bar{y}$ (or $\bar{x}$) in this analogy is *from where* the vertical (or horizontal) distance is actually being measured. This concept of a reference axis, or *datum*, while chosen arbitrarily, forms the basis from which all subsequent calculations and measurements are predicated.

By the same token, learning is acquired most effectively when newer materials build on, and overlap with, formerly comprehended concepts and subjects. Moreover, there are also some topics of a descriptive (ie background) nature that, although they may not have been taught previously, are wholly warranted in an early discussion aimed at introducing a new subject. It should be understood that revisiting past or prefatory materials, in all likelihood, will constitute only a very small part of the course as a whole when undertaking the task of teaching a new subject. An instructor that neglects to conduct such preliminary discussions can be responsible for students leaving many unanswered/unresolved/etc questions in the best-case scenario; at worst, the instructor may prohibit those who might otherwise have been capable of succeeding in the subject to being left a step or two behind.

To illustrate the use of teaching-datums in a classroom setting, the authors have intentionally chosen a subject as the model for this article that is usually not required of most engineering students – plate and shell theory. The prerequisites are typically, integral and differential calculus, differential equations, statics and mechanics of materials, which are likely to have been taken by a wide sector of the readership. In particular, the following three teaching-datums are considered: backgrounds in descriptive subjects, pertinent history and operational subjects.

## DESCRIPTIVE SUBJECTS

As humorous as this may seem, the course title – plates and shells – often invokes in people (even true among some engineers!) ideas about the beach or having something to do with arts and crafts. This alone is a compelling reason to begin the course with descriptive teaching-datums. A logical starting point to initiate the discussion is to conduct a brief survey of how structures are classified geometrically, beginning from the most rudimentary of forms and leading to what is actually termed a plate or shell. To this end, the three types of structures described below are highlighted [2].

Bars

The simplest structure is a bar, ie a single member having one dimension significantly larger than the other two. A host of other familiar structural members can be grouped under this broad category, as illustrated in Figure 1.

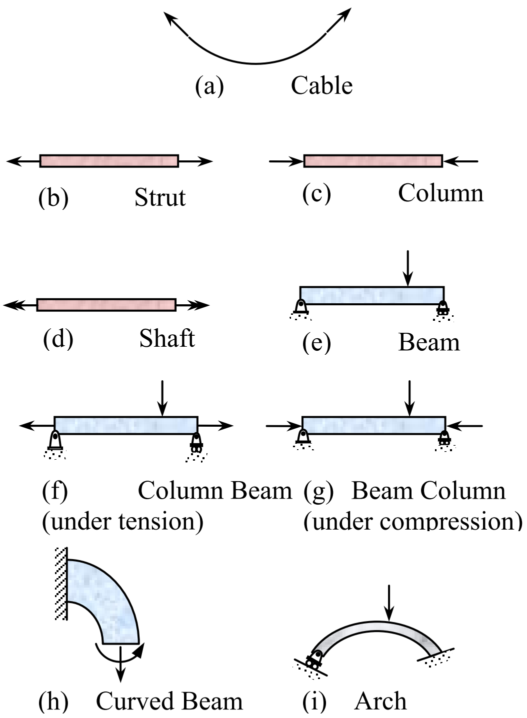


Figure 1: Various types of bars (1a-1i).

Rings

A closed curved member is known as a ring, eg pin-connected eyebar structures (see Figure 2) [3].

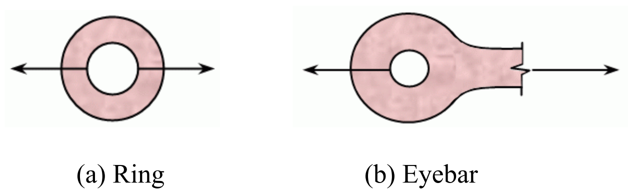


Figure 2: Ring type structures (2a-2b).

Framed Structures

Frames are formed through the assemblage of two or more elements. If, as shown in Figure 3a, the bars are attached by frictionless hinges with members subjected to only axial forces, the configuration is often referred to as a truss; when the members, on the other hand, are rigidly connected, additional loads can also be resisted in addition to those that act axially, and such structures are known simply as frames (Figure 3b).

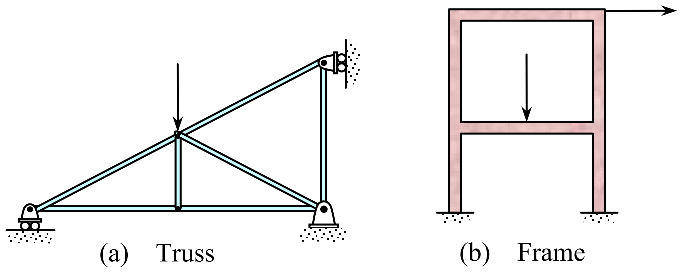


Figure 3: Various framed structures (3a-3b).

Planar or Curved-Planar Structures

Bodies that have two dimensions that are larger relative to the third are referred to as planar or curved-planar structures. *Panels* (Figure 4a) are subjected to in-plane loads, ie forces that are tangential to the two large surfaces. *Plates* (Figure 4b) are similar to panels except that the loads are applied transversely, ie out-of-plane. *Shells* (Figure 4c) may be thought of as curved plates predominantly acting to resist tensile and compressive forces.

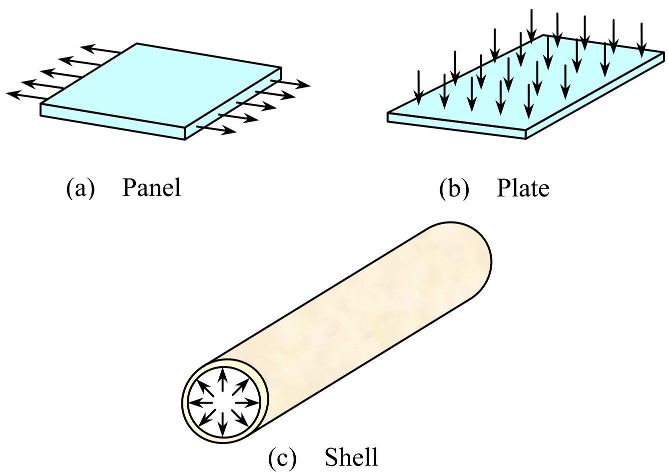


Figure 4: Planar and curved-planar structures (4a-4c).

Plates alone are further distinguished as being thin or thick – depending on the thickness dimension, $t$, relative to the other two dimensions, ie the width, $a$, and length, $b$. Hence, as a general rule, thin plates are defined by an aspect ratio of [4]:

$$\frac{t}{\min\{a,b\}} < \frac{1}{20} \tag{1}$$

Similarly, shells are often defined as being thin when the ratio of thickness, $t$, to the radius of curvature, $r$, is less than $^1/_{20}$ [4]:

$$\frac{t}{r} < \frac{1}{20} \tag{2}$$

Moreover, plates can be classified with respect to deformations. The *small-deflection theory* of plates is considered to prevail whenever the lateral displacement, $w$, in the transverse (out-of-plane) direction is less than half the plate thickness, $t$ [5]:

$$w < \frac{t}{2} \tag{3}$$

This is a noteworthy distinction since the vast majority of plate problems fall within the scope of Equation (3) [6]. Therefore, they warrant the use of the well-known Kirchoff Hypotheses approximations for simplifying the analysis (see eg [7]).

## PERTINENT HISTORY

The development of plate and shell theory has a rich history with contributions being made by well-known mathematicians, scientists and engineers. Because of this, an instructor who jumps right into the heart of this subject without any mention of its pertinent background may actually do students a disservice in their overall learning experience and appreciation of the subject. Indeed, the dissemination of such knowledge constitutes a sensible reference-point for the professor who strives to both rouse enthusiasm among students and impart as complete a treatment of the subject content as possible.

Theoretical developments for most complicated structural systems have typically been initiated by considering static load effects first; however, investigations into plate behaviour began by exploring free vibrations [8]. Leonhard Euler (1707-1783) embarked on such inquiries in 1766, which were further extended by his student and, as it turns out, grandson-in-law, Jacob (II) Bernoulli (1759-1789). In 1809, Napoleon and the French Academy of Science, invited the German physicist, Ernst Florens Friedrich Chladni (1756-1827) to demonstrate patterns produced on small glass plates covered with sand that, when excited by the movement of a violin bow along the edge, caused the granules to propagate until coming to rest along nodal positions, ie non-vibrating sites [9]. The French were so impressed with Chladni's work that they initiated a research competition on plate vibrations with an award of 3,000 francs to be given to the individual who could correctly describe this physical phenomenon mathematically. Several years later, in 1815, the prize was presented to French mathematician, Sophie Germain (1776-1831), who happened to be the only applicant daring enough to take the challenge. While Germain developed a fourth-order partial differential equation to describe plate vibrations, she made a mistake in her choice of boundary conditions, which resulted in an expression lacking the warping term. In 1811, the renowned mathematician, Joseph-Louis Lagrange (1736-1813), who was acting as one of the contest-judges, noticed this discrepancy and, therefore, received co-credit in developing the now famous Germain-Lagrange equation:

$$k\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \frac{\partial^2 w}{\partial t^2} = 0 \qquad (4)$$

where the plate deflection, $w$, is a function of both space ($x$ and $y$) and time ($t$); $k$ is a scalar-constant.

The names of many other prominent scientists – of Hall of Fame status – were involved in the development of plate theory as well. Siméon Denis Poisson (1781-1840) and Claude-Louis Navier (1785-1836), for example, conducted further studies to expound upon the Germain-Lagrange equation. In the field of mathematics, the former is noted for his work in partial differential equations, and in the fields of physics and engineering, is credited with the Poisson's ratio, a material property describing the amount by which a body contracts (or expands) in the two lateral directions relative to the amount of stretching that it is subjected to along the third direction; the latter engineer and physicist is famous for his development of relations describing the motion of fluids and gases, the so-called Navier-Stokes equations.

In 1850, the German physicist most famous for his work on electric circuits, Gustav Robert Kirchoff (1824-1887), published an important thesis on the theory of thin plates wherein two basic assumptions were made: initial lines perpendicular to the middle plane of a plate remain straight and perpendicular to the middle plane during bending, and for small deflections, the middle plane of the plate does not exhibit stretching [8]. The British physicist, William Thomson, better known under his knighted name, Lord Kelvin (1824-1907), who is especially distinguished for his work in thermo-dynamics, first demonstrated that torsional moments acting at the edges of plates could be decomposed into shearing forces. Finally, the eminent English mathematician, Augustus Edward Hough Love (1863-1940) postulated several hypotheses to simplify the analysis of shells, known as Love's first approximations in shell theory [10]. This is a brief tour of famous personages, contextually, although the foregoing list is by no means exhaustive.

## OPERATIONAL SUBJECTS

It is commonly held that the typical engineering class is quite demanding, especially with respect to computations – a possible key cause responsible for steering many a well-intentioned student away from pursuing studies in this discipline. While such a notion may tend to deter the beginning learner, the voice of experience coming from faculty or, for that matter, a more seasoned student, should serve to moderate this type of premature and even aggrandised apprehension.

As with the former two topics, ie *descriptive subjects* and *pertinent history*, the content of more advanced courses like that of plates and shells is predicated upon more rudimentary concepts. With such an understanding, the various *operational subjects* (ie *computational*, in this engineering context) that may, at first glance, appear to be new and intimidating in an advanced class as this should, in actuality, not be cause for undue stress and anxiety. A noteworthy effort from the professor to reassure students that the former operational tools required will be revisited during classroom lecture may assuage any prohibitive perceptions and sentiments that students may possess. Statements from professors such as, *It is assumed that you are all familiar with …*, or, *If you have trouble with … then it is your responsibility to go back and review this material*, may unwittingly undermine the morale of students from the very start and impede their potential for success throughout the remainder of the course.

The graduate-level class on plate and shell theory is highly mathematical in nature, although its framework depends almost exclusively on more elementary computational concepts that are commonly required and taught in subjects from the lower-division, undergraduate curriculum. For example, to determine the deflection equation of rectangular plates having simple-supports at two opposing edges and arbitrary boundary conditions for the others (see Figure 5), the well-known Lévy's approach is conveniently used to solve the governing partial differential equation for plates [11]. This is as follows:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D} \qquad (5)$$

where, as with (4), $w$ is the deflection at any location $(x, y)$, $p$ is the load, and $D$ is the flexural rigidity of the plate. For the sake of compactness, the left-hand side of (5) can also be written as:

$$\nabla^4 w = \frac{p}{D} \tag{6}$$

where, $\nabla^4$ is the biharmonic operator. Similar to the procedure used with ordinary differential equations – a process familiar to the undergraduate student – the total deflection, $w$, is obtained by summing the homogeneous, $w_h$, and particular, $w_p$, solutions:

$$w = w_h + w_p \tag{7}$$

for which Lévy assumed $w_h$ to be of the form:

$$w_h = \sum_{m=1}^{\infty} f_m(y) \sin\left(\frac{m\pi x}{a}\right) \tag{8}$$

with a function, $f_m(y)$, fulfilling support conditions at $y = \pm b/2$.

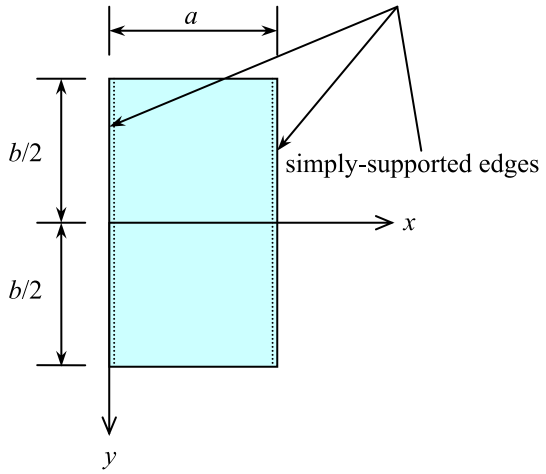


Figure 5: Rectangular plate of dimensions $a \times b$.

Now application of the biharmonic on the homogeneous solution, $\nabla^4 w_h = 0$, gives:

$$\sum_{m=1}^{\infty}\left[\frac{d^4 f_m}{dy^4} - 2\left(\frac{m\pi}{a}\right)\frac{d^2 f_m}{dy^2} + \left(\frac{m\pi}{a}\right)^4 f_m\right]\sin\left(\frac{m\pi x}{a}\right) = 0 \tag{9}$$

which must be true for every value of $x$; however, as there exist values of $x$ such that $\sin(m\pi x/a) \neq 0$ (eg those not belonging to the set: $\{0, \pm a, \pm 2a, \ldots\}$), it, therefore, must be the case that:

$$\frac{d^4 f_m}{dy^4} - 2\left(\frac{m\pi}{a}\right)\frac{d^2 f_m}{dy^2} + \left(\frac{m\pi}{a}\right)^4 f_m = 0 \tag{10}$$

The solution to Equation (10) as taught in an ordinary differential equations class is found by assuming $f_m(y)$ as:

$$f_m(y) = e^{\lambda_m y} \tag{11}$$

and then differentiating it appropriately, from which a characteristic equation results – in this case:

$$\lambda_m^4 - 2\left(\frac{m\pi}{a}\right)^2 \lambda_m^2 + \left(\frac{m\pi}{a}\right)^4 = 0 \tag{12}$$

which finally renders the following general solution:

$$f_m(y) = A_m e^{\gamma_m y} + B_m e^{-\gamma_m y} + C_m y e^{\gamma_m y} + D_m y e^{-\gamma_m y} \tag{13}$$

where $A_m$, $B_m$, $C_m$, and $D_m$ are constants, and $\gamma_m = m\pi/a$.

Apart from a review of ordinary differential equations, other operational subjects that would be appropriate for the professor to revisit before delving headlong into a plates and shells class may include: Taylor series, Fourier series, variational calculus and energy principles, numerical methods (eg's Rayleigh-Ritz solutions, and finite differences) and differential geometry.

## CONCLUSIONS

Borrowing from the idea of reference points, three devices were proposed in the foregoing discussion to promote engineering instruction to be both more receptive to students and to proceed more seamlessly in the new presentation of new subject matters by framing it under formerly learned concepts. These so called *teaching-datums* included: descriptive subjects, pertinent history and operational subjects.

In order to facilitate the discussion, the highly mathematical course on plate and shell theory was selected as a model for this inquiry. It was argued that following proper construction of more advanced courses by the implementation of such strategies enables students to overcome initial intimidation and to also appreciate the efforts of those that came before them, thus promoting their overall comprehension of the subject matter.

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